

System Design Issues for Virtual Reality Simulations with Haptic Displays

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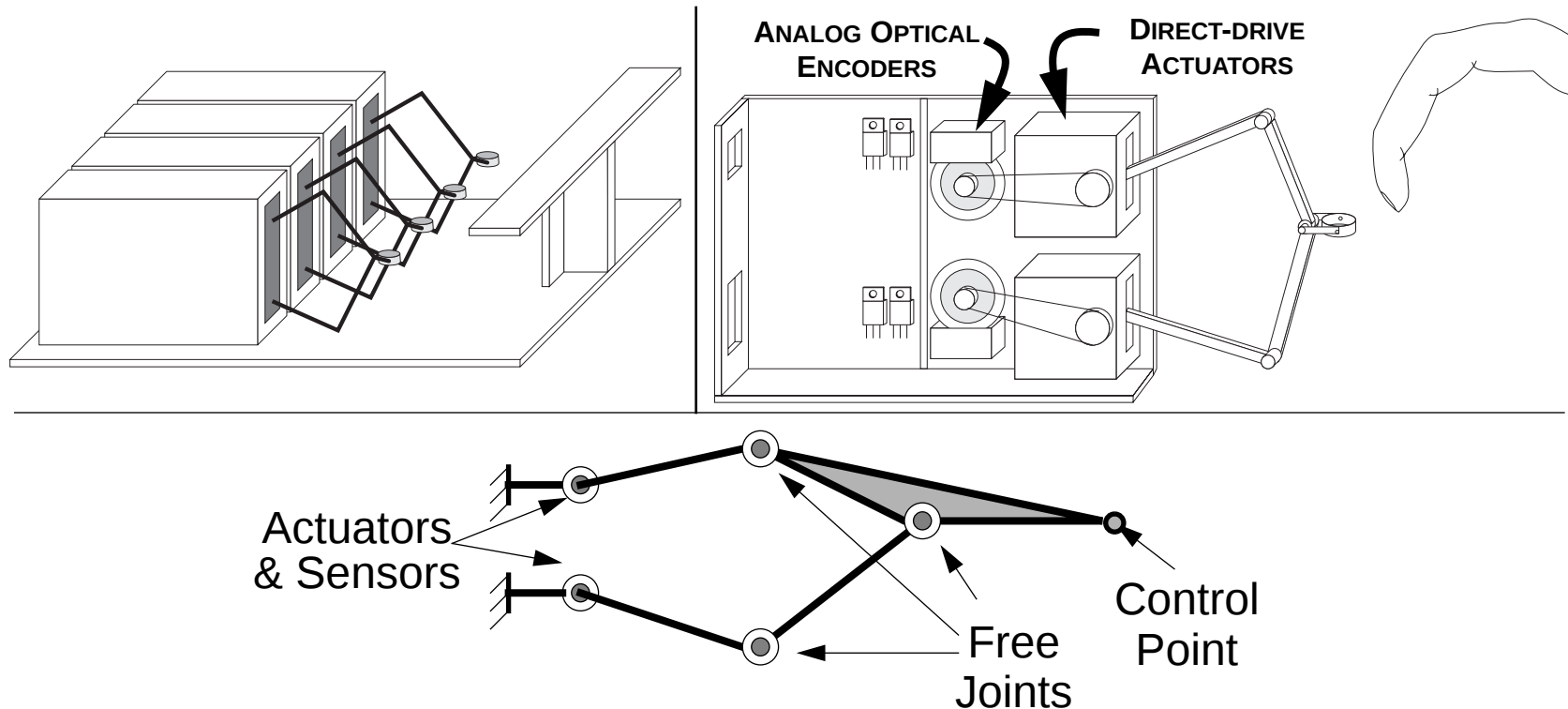
UW Fingertip Haptic Display (FHD)

Research Goal:

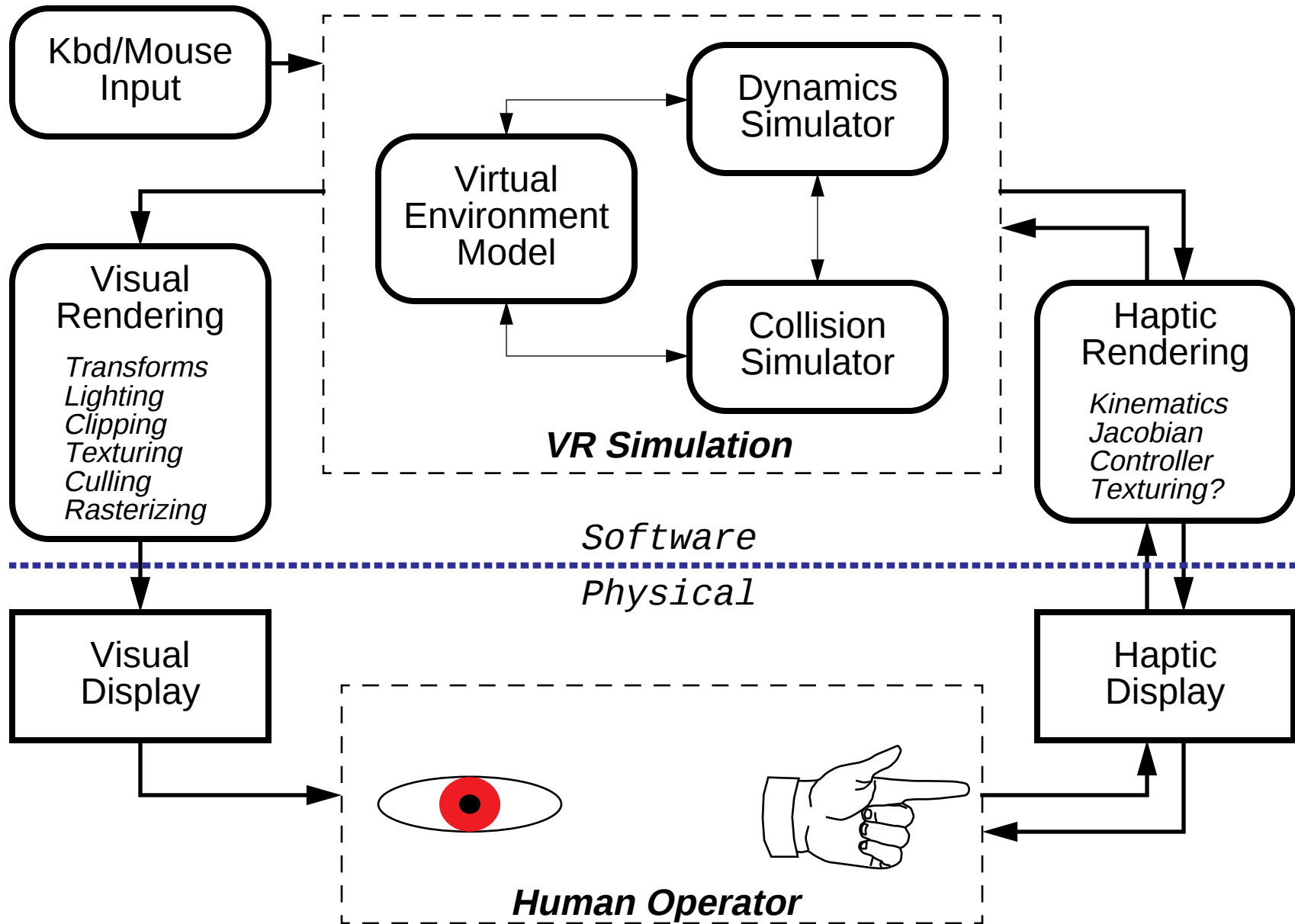
- Explore the utility of a FHD in the detection of surface shapes and contours

Implementation

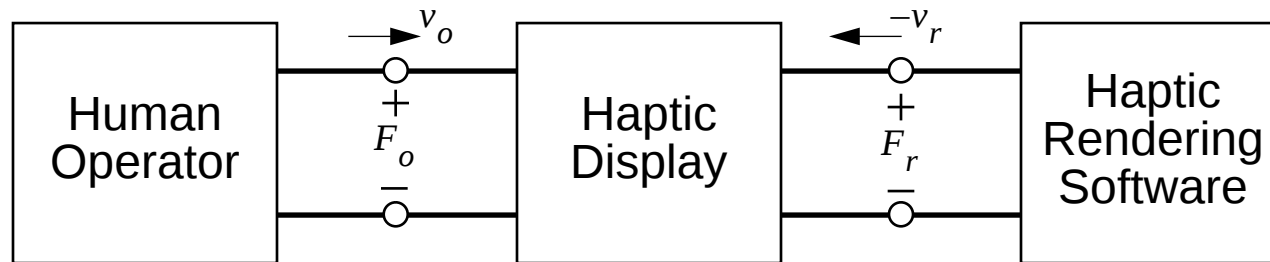
- Flexion/extension motion of the human finger
- Mechanical design to allow multiple FHD's to be used for multiple fingers



- Project support: Ford Motor Company., Intel. Corp.



Two-port haptic display model¹



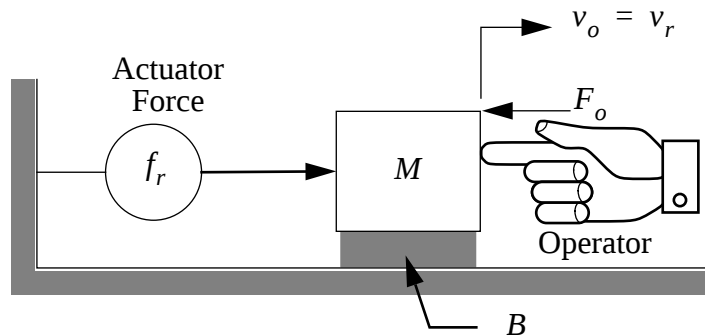
Hybrid Matrix, H satisfies:

$$\begin{bmatrix} F_o \\ -v_r \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} v_o \\ F_r \end{bmatrix}$$

The ideal “transparent”
haptic display has H :

$$\begin{bmatrix} F_o \\ -v_r \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} v_o \\ F_r \end{bmatrix}$$

Simplified 1-axis haptic device model:

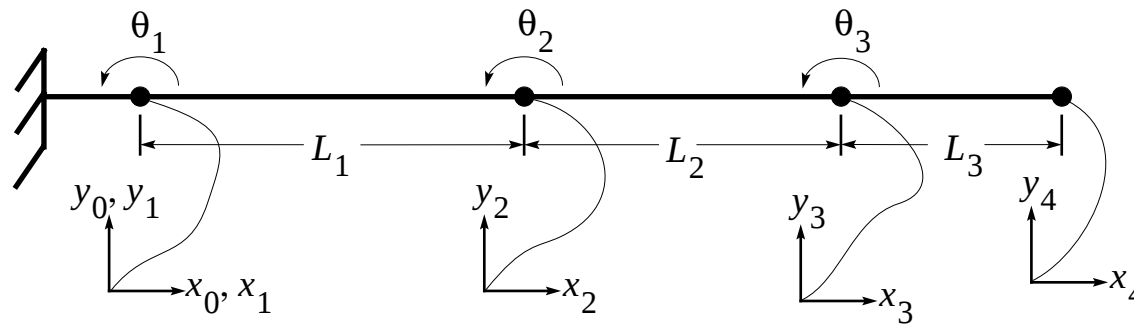
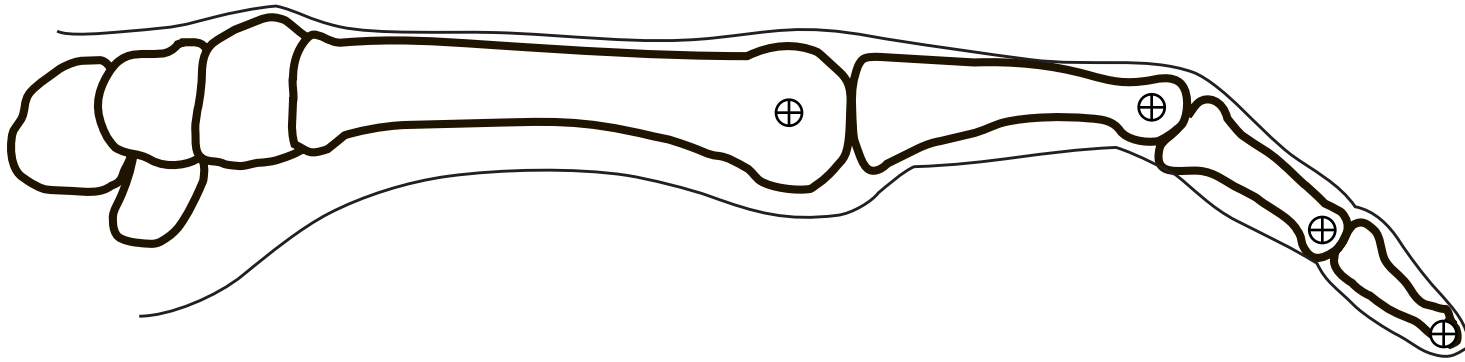


$$F_o = F_r + (B \cdot v_o) + (M \cdot \dot{v}_o)$$

$$\begin{bmatrix} F_o \\ -v_r \end{bmatrix} = \begin{bmatrix} Ms + b & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} v_o \\ F_r \end{bmatrix}$$

1. Adams & Hannaford, 1998.

Kinematic Model of Finger Flexion/Extension



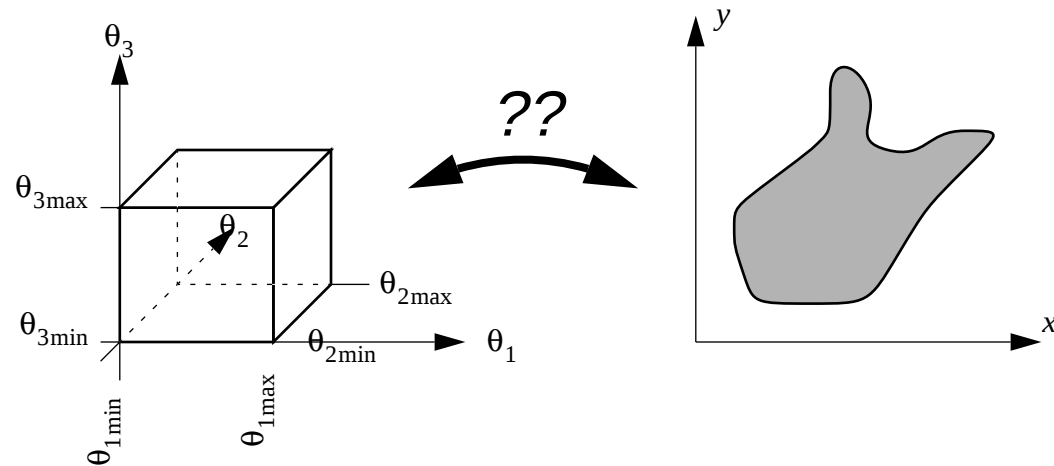
Kinematic
Parameters

$$\begin{bmatrix} L_1 & \theta_{1\text{MIN}} & \theta_{1\text{MAX}} \\ L_2 & \theta_{2\text{MIN}} & \theta_{2\text{MAX}} \\ L_3 & \theta_{3\text{MIN}} & \theta_{3\text{MAX}} \end{bmatrix}$$

Kinematic
Variables

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

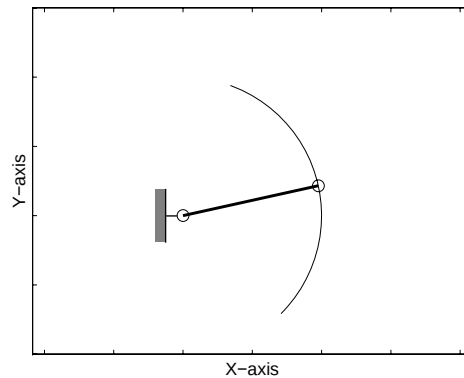
Reachable Workspace:



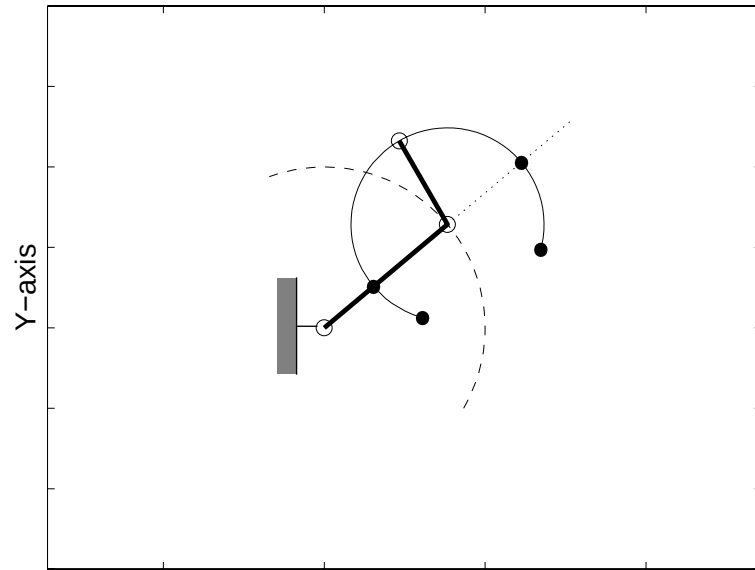
Boundary Representation

- Geometric boundary representation of reachable workspace
- For a serial mechanism with revolute joints moving in a plane, the boundary will be a series of connected arcs

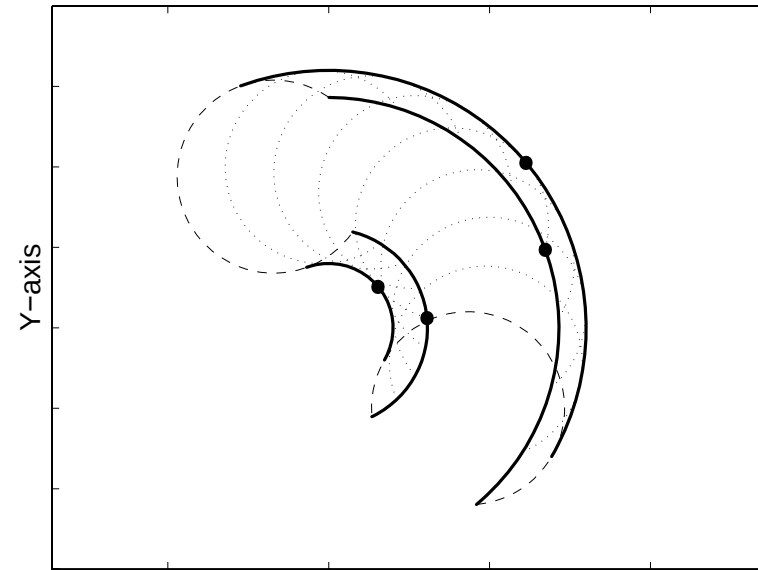
1-DOF Reachable workspace boundary:



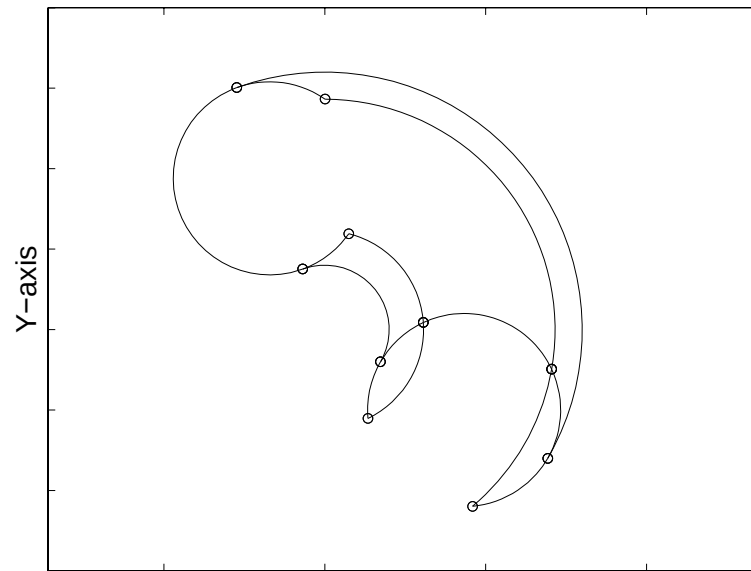
2-DOF reachable workspace boundary



X-axis

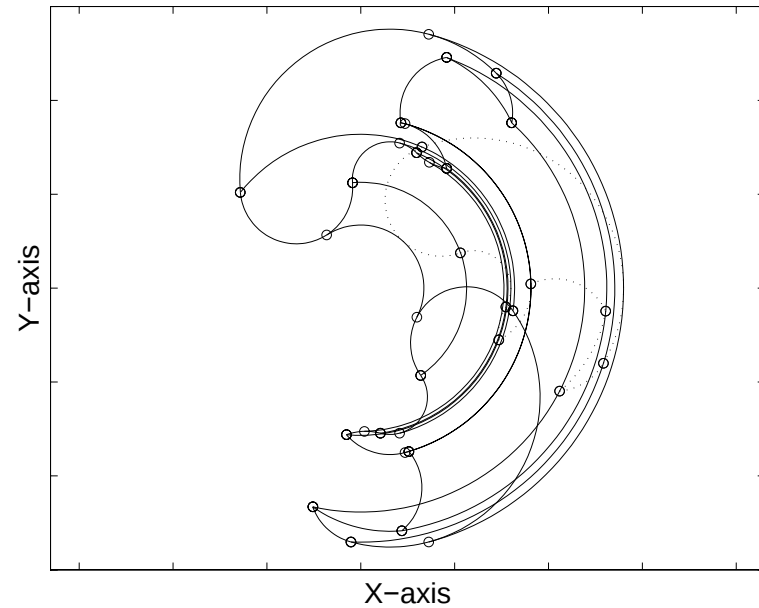
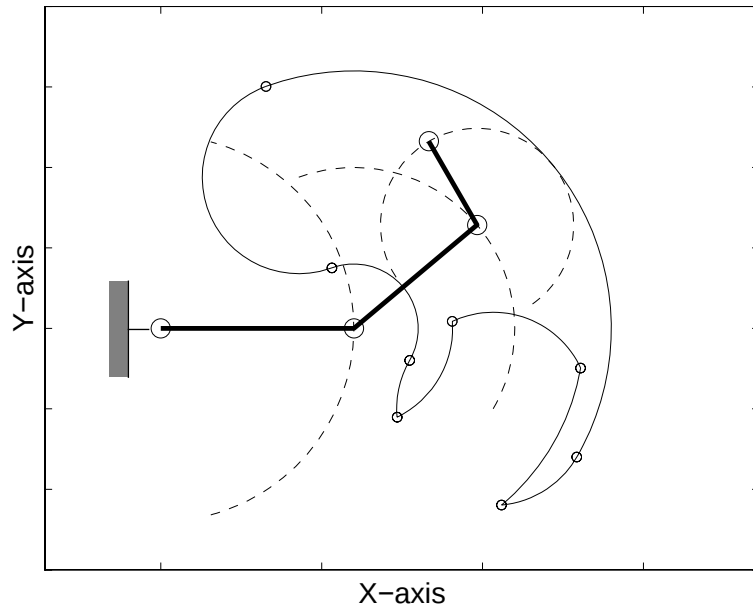


X-axis

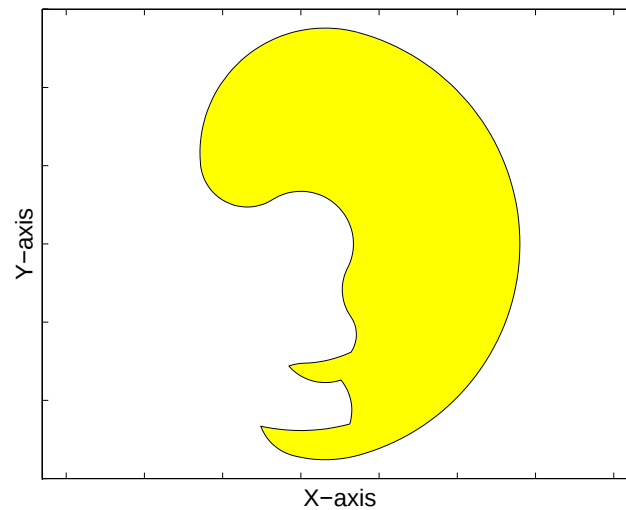


X-axis

3-DOF Reachable Workspace:



- Resulting Reachable Workspace:

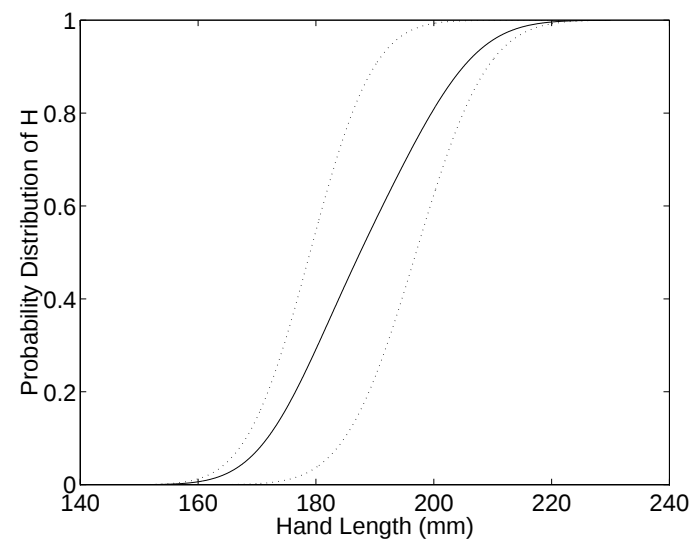
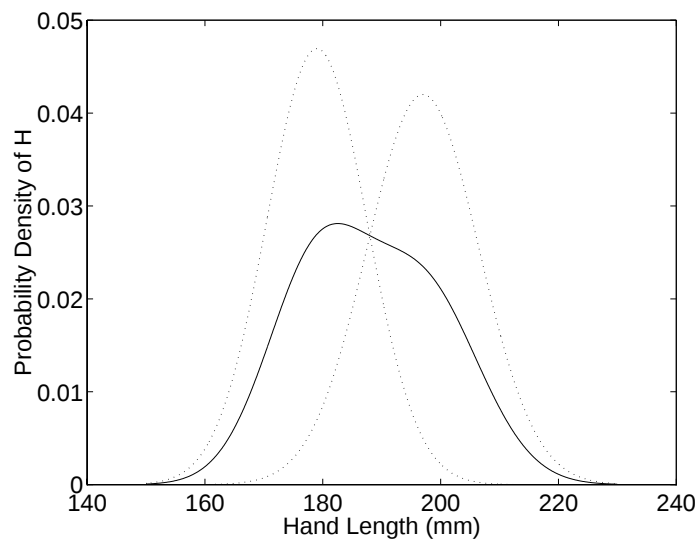


Stochastic Kinematics

- Previous reachable workspace derivation assumes *known* kinematics
- Human hands/fingers vary in size
- Stochastic model of hand length:

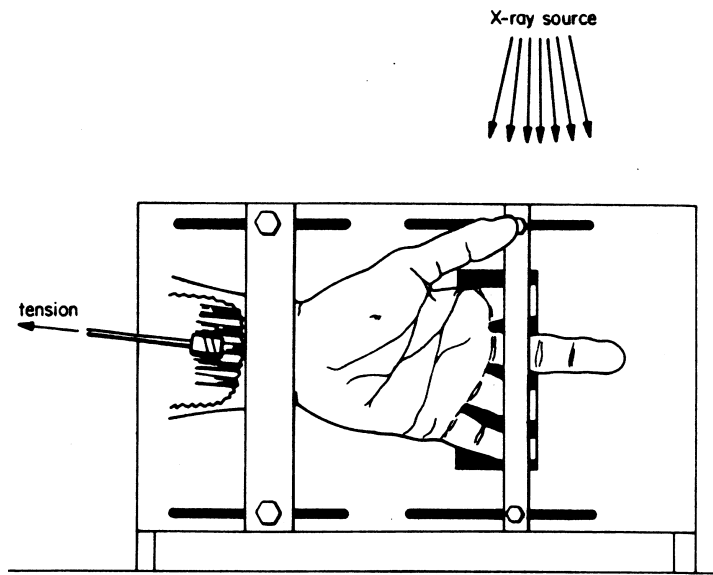
(Adapted from tabular data published in Garrett, 1971)

$$\rho(H) = \frac{\exp\left[\frac{(h-197)^2}{-2(9.5)^2}\right]}{2\sqrt{2\pi(9.5)^2}} + \frac{\exp\left[\frac{(h-179)^2}{-2(8.5)^2}\right]}{2\sqrt{2\pi(8.5)^2}}$$



Finger segment length data

- Buchholz data [1992]
(Hands from 6 cadavers with a “broad spectrum of hand sizes”)
- Correlate hand length with finger segment lengths:



Finger		Proximal Phalanx		Middle Phalanx		Distal Phalanx	
Name	#	Coeff	Error	Coeff	Error	Coeff	Error
First	1	0.245	0.001	0.143	0.003	0.097	0.002
Middle	2	0.266	0.003	0.170	0.003	0.108	0.003
Ring	3	0.244	0.003	0.165	0.002	0.107	0.004
Little	4	0.204	0.002	0.117	0.002	0.093	0.003

- So the link lengths of finger 1 (index finger) of with average hand length would be:

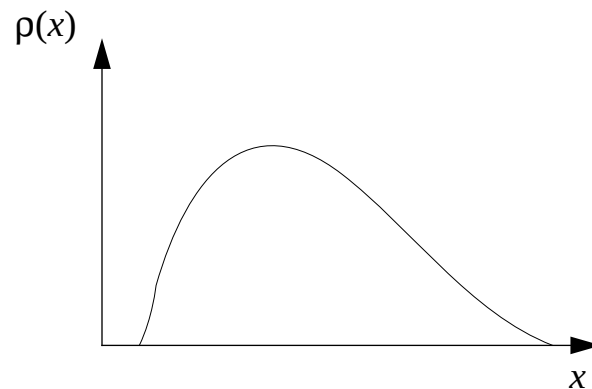
$$\begin{aligned}
 \begin{bmatrix} l_1 & l_2 & l_3 \end{bmatrix} &= 197\text{mm} \times \begin{bmatrix} 0.245 & 0.143 & 0.097 \end{bmatrix} \\
 &= \begin{bmatrix} 48.3 & 28.2 & 19.11 \end{bmatrix} \text{mm}
 \end{aligned}$$

Stochastic Reachable Workspace:

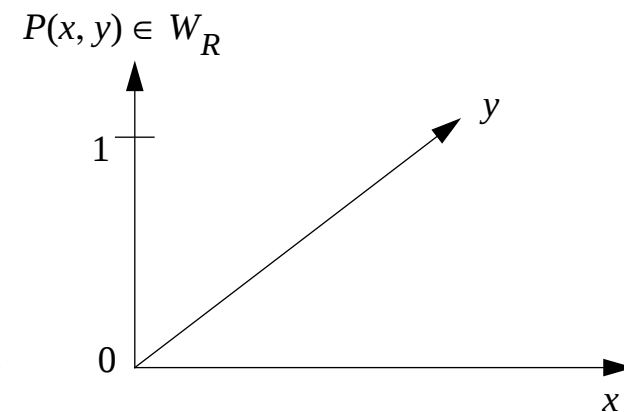
Kinematics

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} \quad \begin{bmatrix} L_1 & \theta_{1\text{MIN}} & \theta_{1\text{MAX}} \\ L_2 & \theta_{2\text{MIN}} & \theta_{2\text{MAX}} \\ L_3 & \theta_{3\text{MIN}} & \theta_{3\text{MAX}} \end{bmatrix}$$

Stochastic Model



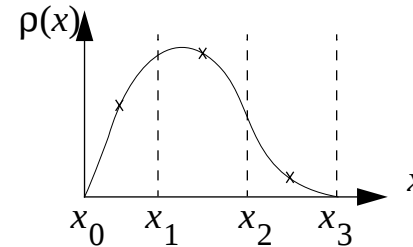
Stochastic Reachable Workspace



Stochastic Workspace Computation

- Probability density and distribution are related by:

$$P(x_1 < x \leq x_2) = \int_{x_1}^{x_2} p(x) dx$$



(1)

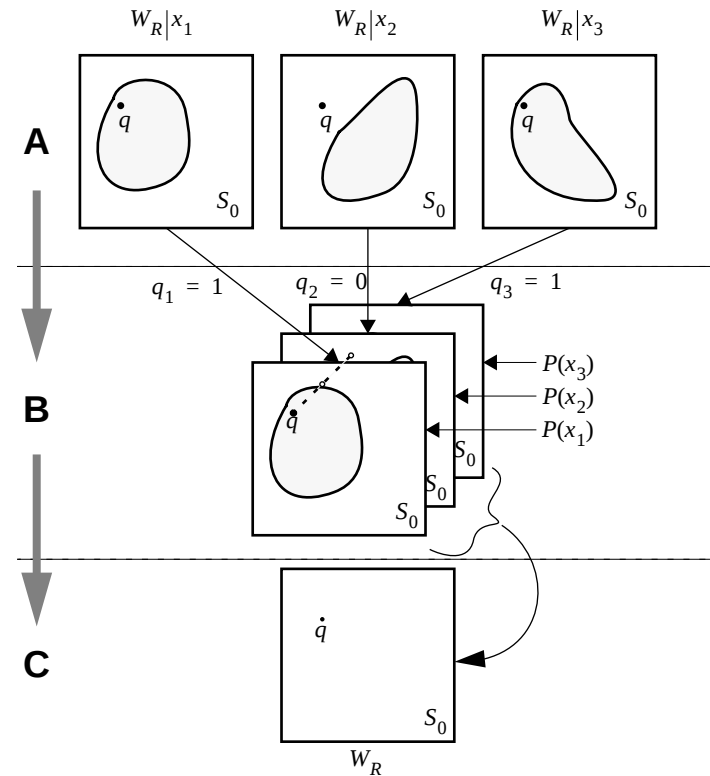
- Piecewise integration of reachable workspace:

$$\begin{aligned} P(q) &= P(q|x_1)P(x_1) + P(q|x_2)P(x_2) + P(q|x_3)P(x_3) \\ &= (1)P(x_1) + (0)P(x_2) + (1)P(x_3) \\ &= P(x_1) + P(x_3) \end{aligned}$$

Since $\sum_i P(x_i) \equiv 1$

and $P(q|x_i) \in \{0, 1\} \quad \forall i$

then $0 \leq P(q) \leq 1$ must be true



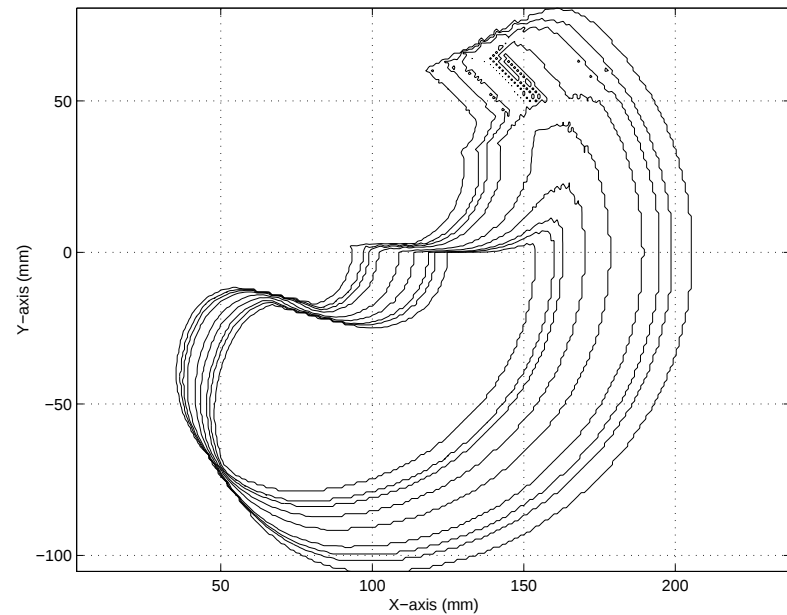
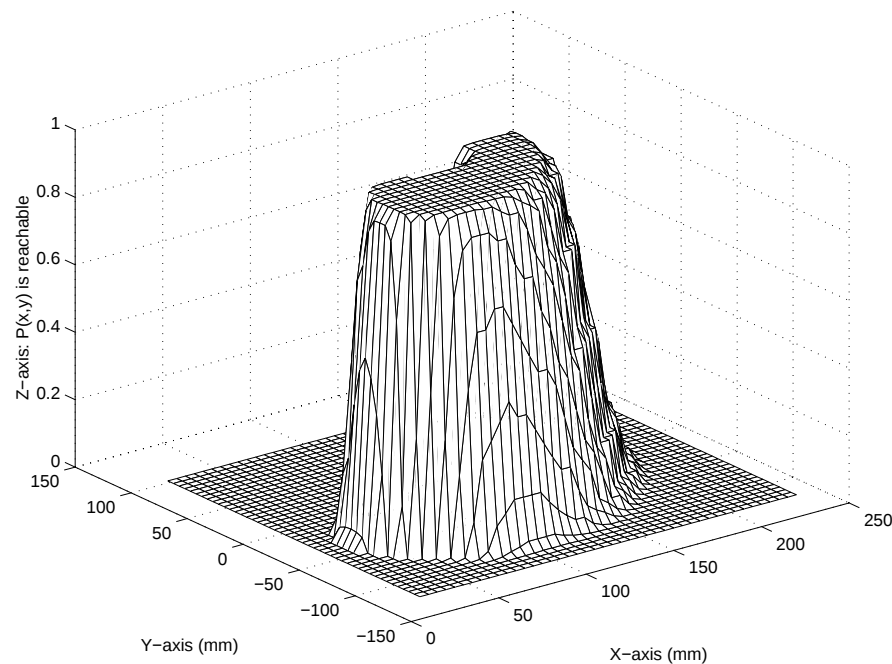
Results: Index Finger Stochastic reachable workspace

- Assume Joint motion ranges of

$$-90^\circ \leq \theta_1 \leq 45^\circ$$

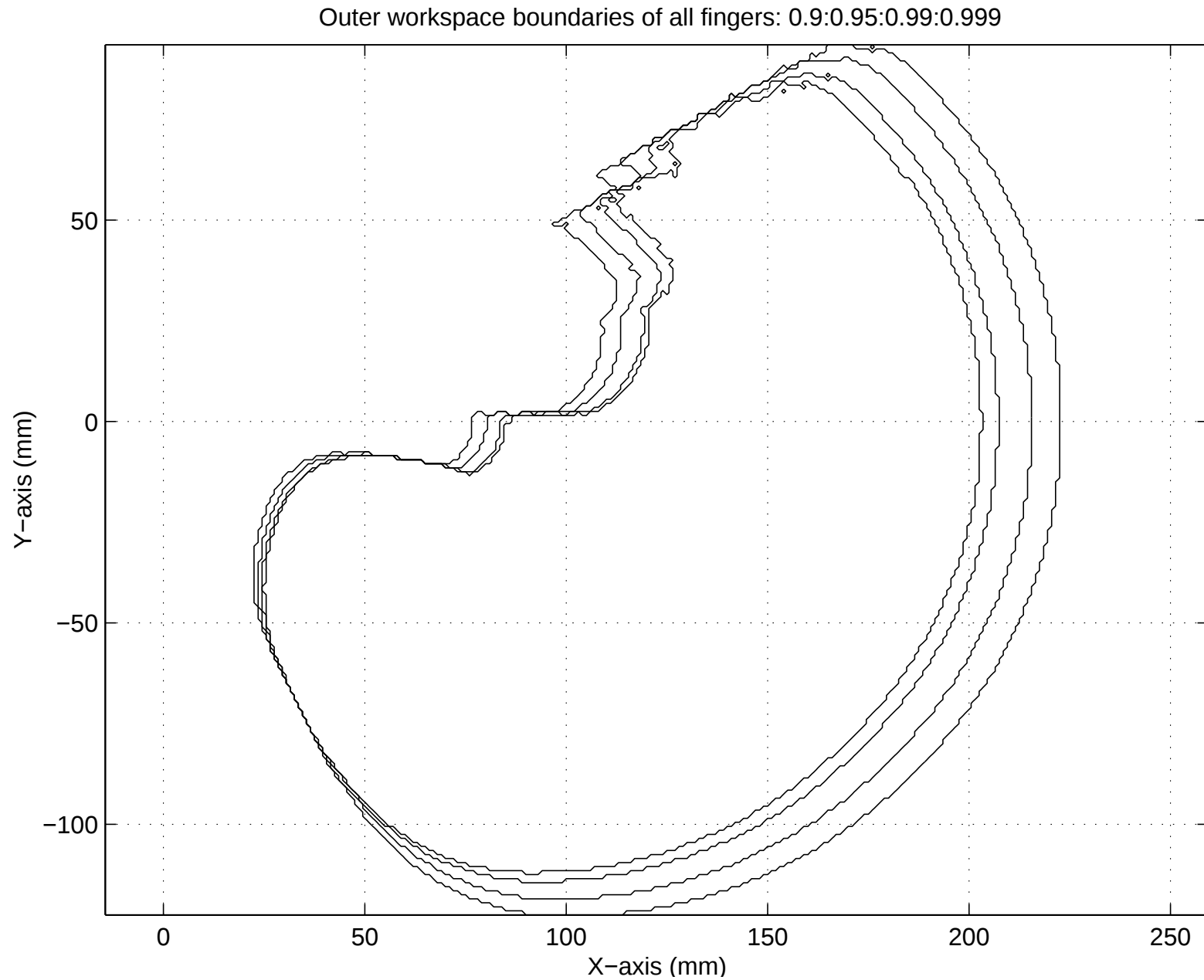
$$-120^\circ \leq \theta_2 \leq 0^\circ$$

$$-90^\circ \leq \theta_3 \leq 50^\circ$$



- Contours are at $P(q) = \{0.99, 0.95, 0.90, 0.75, 0.50, 0.20, 0.10, 0.05, 0.01\}$

Results: Boolean combinations of all fingers

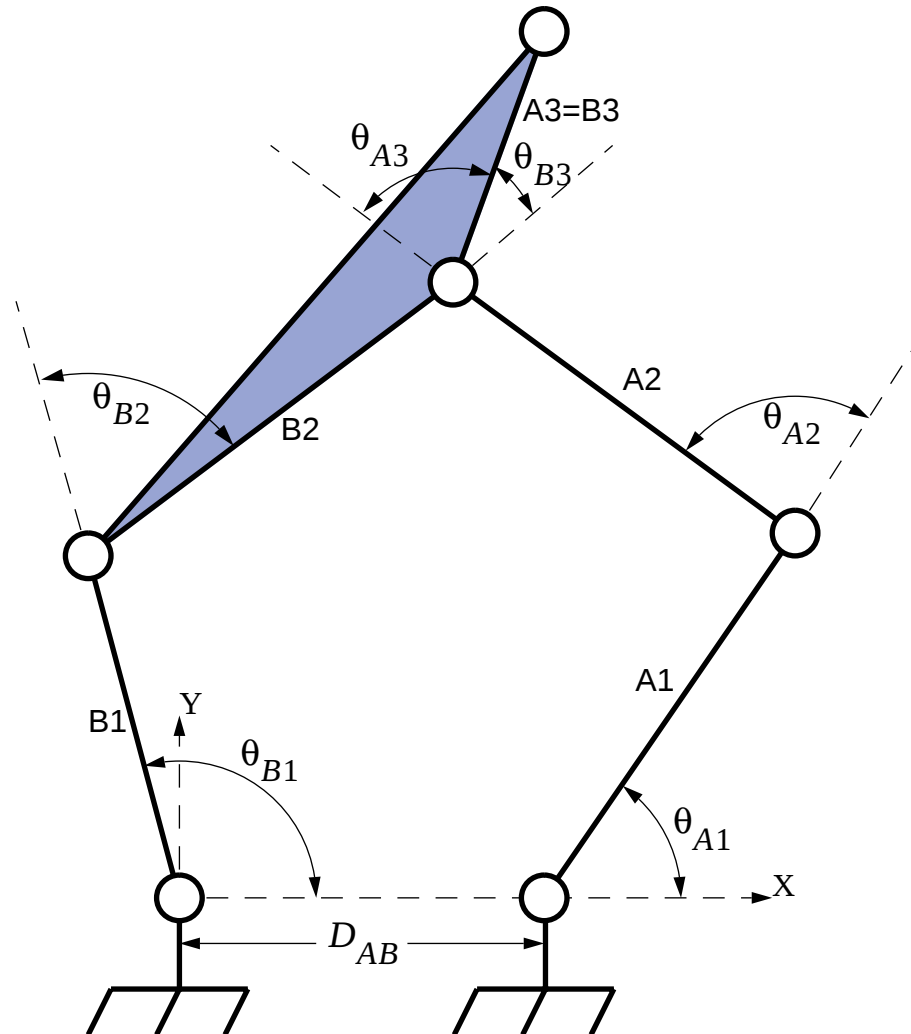


FHD Workspace Design

- Goal: *Just-big-enough* design for accommodating the human finger
 - Lower mass, higher force output (smaller moment arm), higher stiffness
- Extended 5-bar Kinematics and parameters:

Design Parameters

Length	Motion Range	
	Min	Max
A1	$\theta_{A1\min}$	$\theta_{A1\max}$
A2	$\theta_{A2\min}$	$\theta_{A2\max}$
A3	$\theta_{A3\min}$	$\theta_{A3\max}$
B1	$\theta_{B1\min}$	$\theta_{B1\max}$
B2	$\theta_{B2\min}$	$\theta_{B2\max}$
B3	θ_{B3}	
D_{AB}		



Optimal FHD Design Approach

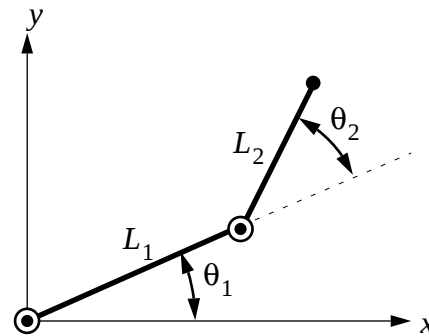
1. Discretize the design space (5-bar kinematic parameters)
2. Evaluate each possible candidate mechanism's workspace
3. Choose “best” mechanism

Optimization Goals

- Maximize force output
- Maximize reachable workspace
- Provide “good” force isotropy

Review

Given 2-link serial mechanism with planar revolute joints:



Position & Velocity Relations:

$$x = L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2)$$

$$y = L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + \theta_2)$$

$$\theta_2 = \pm \arccos\left(\frac{x^2 + y^2 - L_1^2 - L_2^2}{2L_1L_2}\right)$$

$$\theta_1 = \operatorname{atan2}\left(\frac{y}{x}\right) \mp \arccos\left(\frac{x^2 + y^2 + L_1^2 - L_2^2}{2L_1\sqrt{x^2 + y^2}}\right)$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = J(\theta)\dot{\theta}$$

$$J(\theta) = \begin{bmatrix} -L_1 \sin(\theta_1) - L_2 \sin(\theta_1 + \theta_2) & -L_2 \sin(\theta_1 + \theta_2) \\ L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2) & L_2 \cos(\theta_1 + \theta_2) \end{bmatrix}$$

Applications of the Jacobian:

- Velocity Controller: $\dot{\theta} = J^{-1}(\theta)\dot{X}$ (singularities!)
- Force Controller: $\tau = J^T(\theta)F$

Singularity analysis

- Singularities in $J(\theta)$ occur as a function of θ and define the positions at which the mechanism loses its ability to move in one or more cartesian directions.
- The Jacobian is singular when $\det(J(\theta)) = 0$
- Singular values of J:

$$\sigma_i = \sqrt{\lambda_i} \text{ where } \lambda_i \text{ is the } i^{\text{th}} \text{ eigenvalue of } [J^T(\theta)J(\theta)] \quad (2)$$

Uses of singular values

- $\sigma_{\max}(\theta)$ and $\sigma_{\min}(\theta)$ are proportional to the max/min possible Cartesian velocity
- $\frac{1}{\sigma_{\max}(\theta)}$ and $\frac{1}{\sigma_{\min}(\theta)}$ are proportional to the min/max possible Cartesian force
- $\frac{\sigma_{\min}}{\sigma_{\max}}(\theta)$ defines the isotropy of velocity/force output

FHD Design Optimization Approach

- Parameter space reduction: 18 parameters reduced to 8 control variables
- Define A_{WR} as the area of the portion of reachable workspace with isotropy ≥ 0.25
- Define $\widehat{F_{\min}}$ as the lowest minimum force ($\frac{1}{\sigma_{\max}(\theta)}$) in the entire workspace, A_{WR}
- Objective Function: $\text{MAX}(A_{WR} \times \widehat{F_{\min}})$

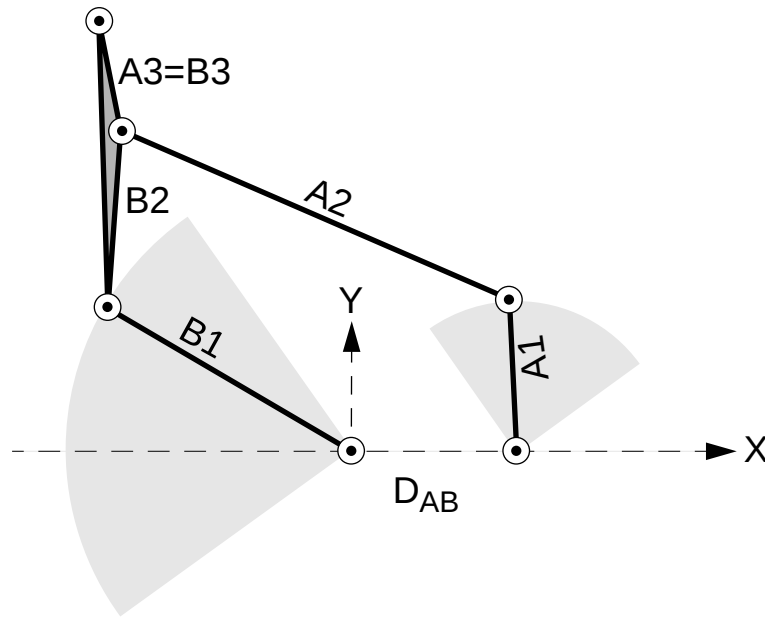
$$J(\theta) = \begin{bmatrix} -B_1 \sin(\theta_{B1}) + \frac{B_1 B_3 \sin(\theta_{B1} + \theta_{B3} + \alpha) \sin(\theta_{B1} - \theta_{A1} - \theta_{A2})}{B_2 \sin(\theta_{B1} + \theta_{B2} - \theta_{A1} - \theta_{A2})} & \frac{A_1 B_3 \sin(\theta_{B1} + \theta_{B3} + \alpha) \sin(\theta_{A2})}{B_2 \sin(\theta_{B1} + \theta_{B2} - \theta_{A1} - \theta_{A2})} \\ B_1 \cos(\theta_{B1}) - \frac{B_1 B_3 \cos(\theta_{B1} + \theta_{B3} + \alpha) \sin(\theta_{B1} - \theta_{A1} - \theta_{A2})}{B_2 \sin(\theta_{B1} + \theta_{B2} - \theta_{A1} - \theta_{A2})} & \frac{-A_1 B_3 \cos(\theta_{B1} + \theta_{B3} + \alpha) \sin(\theta_{A2})}{B_2 \sin(\theta_{B1} + \theta_{B2} - \theta_{A1} - \theta_{A2})} \end{bmatrix} \quad (3)$$

$$F_{\min} = \left[0.5 \left(J_{11}^2 + J_{12}^2 + J_{21}^2 + J_{22}^2 \right) + \sqrt{0.25 \left(J_{11}^2 + J_{21}^2 - J_{12}^2 - J_{22}^2 \right)^2 + \left(J_{11}^2 J_{12}^2 + J_{21}^2 J_{22}^2 \right)^2} \right]^{-\frac{1}{2}} \quad (4)$$

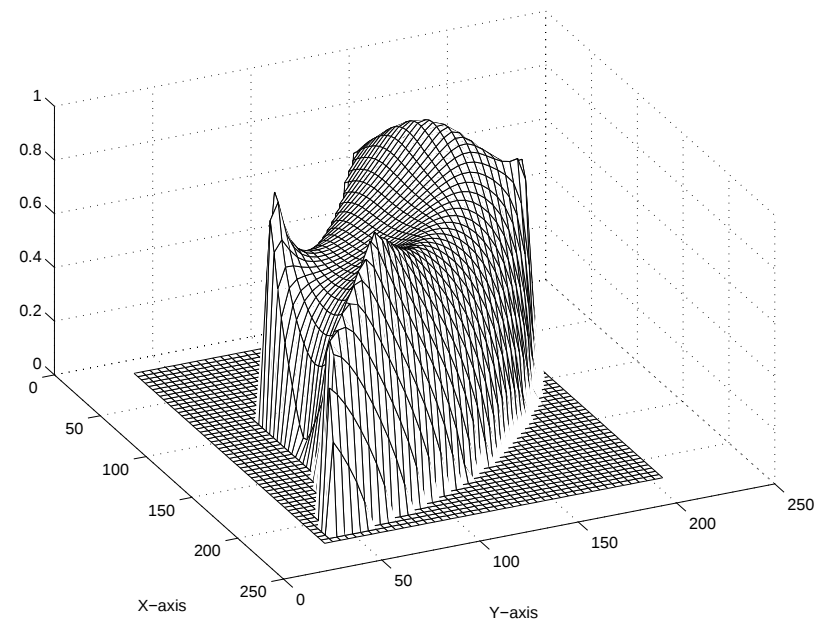
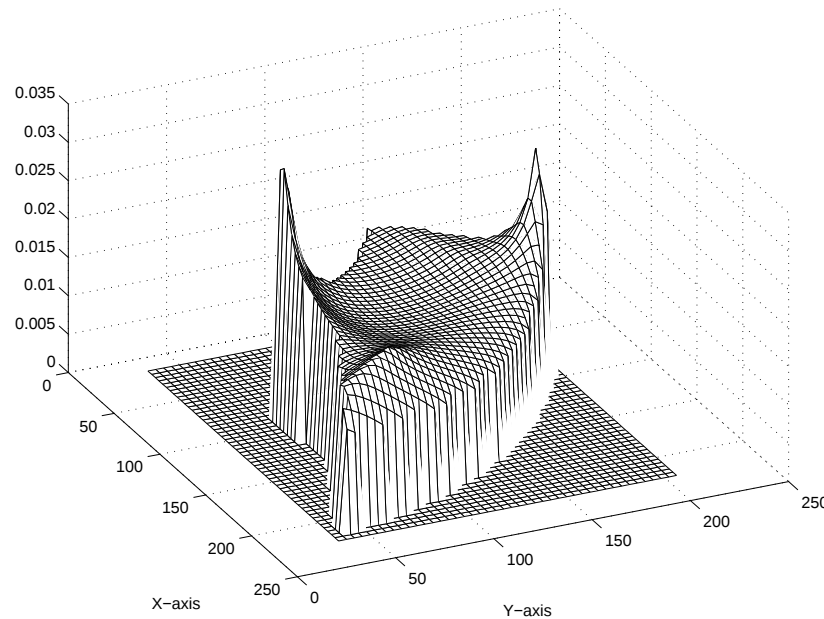
$$ITP = \frac{F_{\min}}{\left(F_{\min} = \left[0.5 \left(J_{11}^2 + J_{12}^2 + J_{21}^2 + J_{22}^2 \right) - \sqrt{0.25 \left(J_{11}^2 + J_{21}^2 - J_{12}^2 - J_{22}^2 \right)^2 + \left(J_{11}^2 J_{12}^2 + J_{21}^2 J_{22}^2 \right)^2} \right]^{-\frac{1}{2}} \right)} \quad (5)$$

- Optimization process was divided among ~15 CPU's
- Runtime was ~48 hours

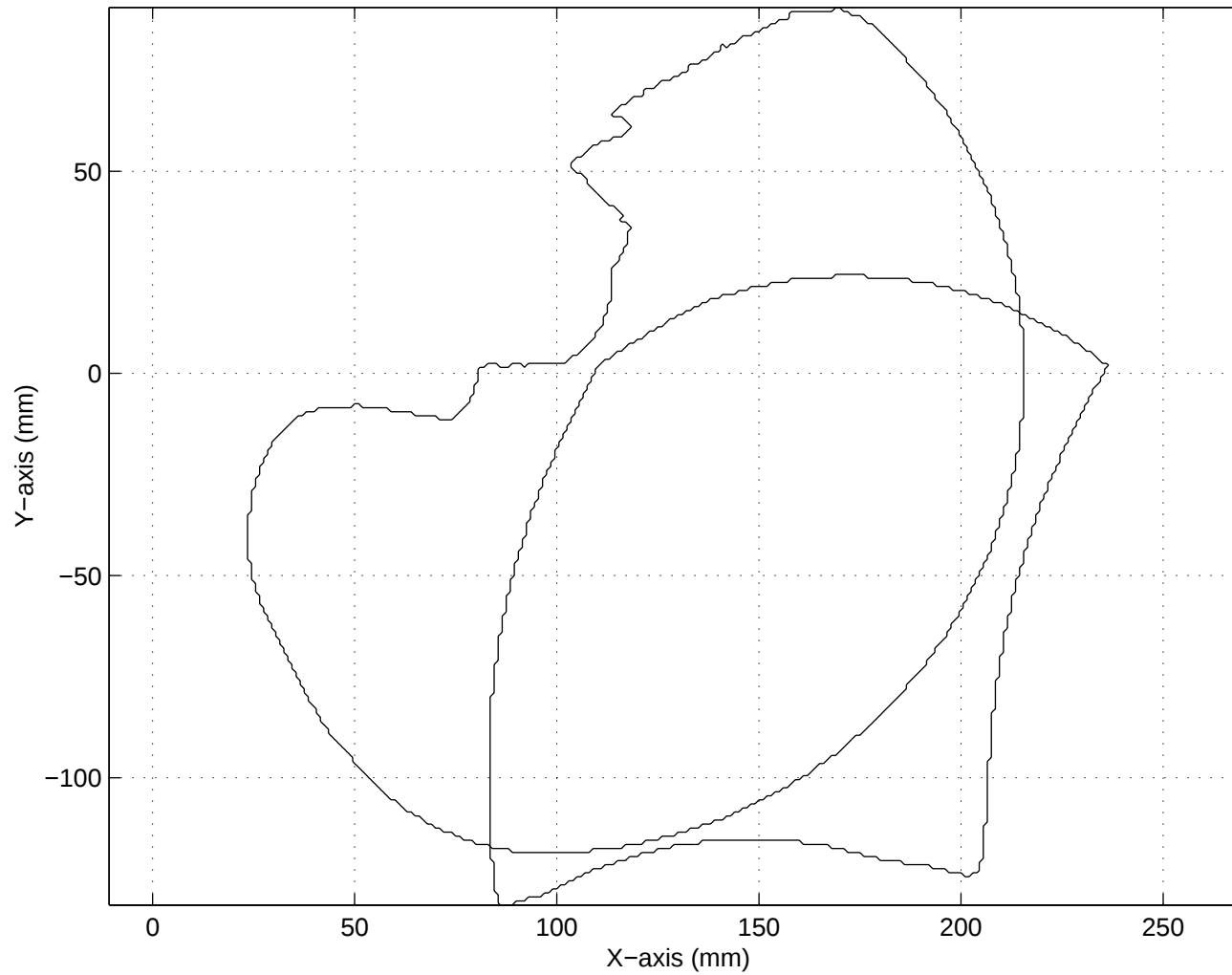
Kinematic Optimization Results



Link	Len (mm)	Min (Deg)	Max (Deg)
A1	53	35	125
A2	150	0	160
A3	40	-160	-5
B1	100	125	215
B2	63	-160	0
B3	40	15	
Dab	58	—	



Comparison of finger and FHD workspaces:



FHD computing environment

- Solaris 2.6-x86 running on 200MHz quad PentiumPro
 - Dell server with 512MB RAM, 0.5MB Cache/chip, 9GB UltraSCSI disk
- Tech80 ISA card with
 - On-board timer/interrupt controller
 - On-board 48-line digital I/O port
 - Four IP daughter module slots
- Encoder IP Module
 - Supports 6 encoder axis
 - Programmable digital filters for encoder inputs
 - Supports wrap-around interrupts, and error detection interrupts
- DAC IP Module
 - Eight 12-bit DAC channels
- Driver
 - open()/close() routines
 - mmap() routine to map kernel pages for Encoder/DAC I/O
 - ioctl() routine to configure interrupt timer, DIO, encoders and DAC, etc.
- Results so far:
 - Interrupt latency 10-24us (12us mean, 3us sdev)

“Hydra” Graphics software:

- 1989-90: Implemented at JPL as an SGI GL application on SGI IRIS-4D system
 - Used for predictive and preview displays, video overlays
- Loads geometry from data file at start-up
- Supports interactive geometry modification via network connections
- Supports high speed joint motion updates via shared memory
- UW:1991-97: Used for design visualization and debugging of kinematics software

Current activities:

- VRML import from MCAD system
- Port from GL to OpenGL
- Add in basic dynamics/collision modules

Problems with using Solaris Unix

- Scheduler
- Interrupt<->Processor assignment
- Direct thread scheduling control desirable
- OpenGL support
- Cache coherency issues (SMP)

Computing Environment Issues

- Real-time computing environment
- Model format (representation)
- CAD/VR Data interchange (including haptic properties)
- Process partitioning (both simulation and rendering)
- Scaling: SMP and/or network interconnection
- Trade-off of speed vs. accuracy
- Price

Alternative OS environments:

- RT-Linux, QNX/Neutrino, VxWorks, RT-NT?